Fatigue Criterion to System Design, Life, and Reliability

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A generalized methodology to structural life prediction, design, and reliability based on a fatigue criterion is advanced. The life prediction methodology is based in part on work of W. Weibull and of G. Lundberg and A. Palmgren. The approach incorporates the computed life of elemental stress volumes of a complex machine element to predict system life. The results of coupon fatigue testing can be incorporated into the analysis, allowing for life-prediction and component- or structural-renewal rates with resonable statistical certainty.

Nomenclature

a	= life-adjustment factor
B	= material constant, N/m$^{1.979}$ (lb/in.$^{1.979}$)
C	= dynamic load capacity, N (lb)
c	= stress-life exponent
e	= Weibull exponent or slope, $1/\alpha$
F	= probability of failure
f	= failure probability density function
h	= exponent
k	= component load cycles per input shaft revolution
L	= life, hours, input shaft revolutions, or stress cycles
L_A	= adjusted life at a 90% probability of survival, hours, input shaft revolutions, or stress cycles
L_m	= mission life, hours, input shaft revolutions, or stress cycles
L_{10}	= 10% life, life at which 90% of a population will survive, hours, input shaft revolutions, or stress cycles
l	= involute length, m (in.)
m	= number of planets
N	= number of gear teeth
n	= load-life exponent
P	= applied load, N (lb)
$P(x)$	= probability for occurrence of event x
R	= cumulative renewal function
R_1, R_2	= radii of curvature of pinion and gear, respectively, m (in.)
r	= renewal density function
S	= probability of survival
S_T	= system probability of survival
T	= dummy variable of integration
t	= time or time function
V	= stressed volume, m^3 (in.^{3})
w	= face width of gear tooth, m (in.)
X	= time function or stress function
X_β	= characteristic strength, N/m^2 (psi)
X_μ	= stress below which no specimens fail, N/m^2 (psi)
z	= depth to critical maximum shearing stress, m (in.)
α	= shape parameter
β	= characteristic life, hours, input shaft revolutions, or stress cycles
γ	= mean time to failure, hours, input shaft revolutions, or stress cycles
η	= stress cycles to failure
μ	= location parameter or time below which no failure is expected to occur, hours, input shaft revolutions, or stress cycles
σ_u	= fatigue limit, N/m^2 (psi)
$\Sigma\rho$	= curvature sum, m^{-1} (in.$^{-1}$) $[=(1/R_1)+(1/R_2)]$
τ	= critical stress, N/m^2 (psi)

Subscripts

a	= first load
B	= bearing
$B1$	= front pinion bearing
$B2$	= rear pinion bearing
$B3$	= main drive bearing
$B4$	= carrier support bearing
$B5$	= prop thrust bearing
$B6$	= prop radial bearing
$B7$	= planet bearing
b,c,d	= second, third, and fourth loads, respectively
G	= gear
$G1$	= pinion gear
$G2$	= main drive gear
$G3$	= sun gear
$G4$	= planet gear
$G5$	= ring gear
T	= total gearbox or system
t	= gear tooth
$t1$	= tooth of sun gear-planet gear mesh
$t2$	= tooth of planet gear-ring gear mesh
10	= 90% probability of survival
1	= planet gear meshing with sun gear
2	= planet gear meshing with ring gear

Introduction

DESIGN of machine elements is based for the most part on yield stresses and fatigue-limiting stresses. In addition to the components material properties, proper consideration must be given to the effects of notches, surface condition, component size, residual stress, temperature, duty cycle, and environmental factors, such as corrosive or chemical exposure. For most machine elements, individuals and organizations usually develop design methodology based on engineering fundamentals found in most machine design texts and

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factors based on their corporate experience and test data. As a result, it is not too unusual for different organizations or individuals starting with the same or similar design requirements to reach dissimilar conclusions or designs while seemingly applying the same fundamental engineering principles to the problem.

Setting aside the subjective creative aspects of design, there appears to be nonuniformity of data from which numbers and design factors are selected, as well as differences in the computer codes and boundary conditions used in the design process. To compound these difficulties, fatigue data used to establish fatigue limits are usually of a limited nature, with the conditions under which the data were obtained not adequately defined or reported. Such items as temperature, humidity, number of specimens, specimen size and volume, heat treatment, hardness, surface finish, and life distribution are not given.

The established fatigue limit for much of the reported data is a mean value. From a statistical viewpoint, the median value is equal to or less than the mean. This can be interpreted as meaning that before a fatigue limit is determined, there is a probability that 50% or more of the specimens had failed. In other words, even at the fatigue-limiting stress, life is finite. Of course, experienced design engineers have recognized this for years. They have added safety factors to their design procedure, usually based on experience. While these procedures are generally adequate, they can result in overdesigned, oversized, overweight, and overcost structures.

A fundamental principle of good design is to recognize that any structure can fail. However, should it fail, it should be designed not to cause personal injury or secondary damage. In other words, if the structure fails, it fails in a safe or benign manner. Once a structure is designed to fail in a benign manner, it then can be designed for finite life, whereby the overall size, weight, and cost can be reduced and still meet the reliability requirements of the application.

It therefore becomes the objective of the work reported herein to advance a generalized methodology for structural-life prediction, design, and reliability based on a fatigue criterion. The life-prediction methodology is based in part on the work of W. Weibull[1,2] and G. Lundberg and A. Palmgren.[3-6]

Statistical Method

In the late 1930s (circa 1937) W. Weibull in Sweden attempted to linearize graphically various types of statistical data distributions for small sample sizes. By trial and error, Weibull found that by having $\ln\ln 1/S$ as the ordinate and $\ln X$ as the abscissa, where S is the probability of survival or statistical percent of samples surviving and X is a time function, most engineering data distributions will plot on a straight line.[7] Hence, it became possible for small amounts of data to estimate a generalized population distribution for a population of infinite size. Having empirically determined this, Weibull developed a theoretical basis for what was to become known as the Weibull distribution, or Weibull plot, which was published in 1939.[1] Weibull[8] defines the distribution function as "an adequate expression for a large class of phenomena which have the property that the probability of nonoccurrence of an event is equal to the product of the elementary probabilities,"

$$S=\exp\{-[(X-\mu)\beta]^{1/\alpha}\} \tag{1}$$

The function involves three parameters: α, the shape parameter; β, the scale parameter; and μ, the location parameter. Where the shape parameter α is equal to 1, 0.5, and 0.28, the respective distributions approximated are exponential, Rayleigh, and normal (Gaussian). A typical Weibull plot for rolling-element bearing fatigue data is shown in Fig. 1. The slope of the line "e" which is called the Weibull exponent or slope, is equal to $1/\alpha$. For most rolling-element bearing data, e equals 1.1. The location parameter μ is a finite time under which there would be a zero probability of failure. If failure can occur in one stress cycle, the location parameter μ would be assumed to be zero.

For fatigue data, if stress cycles to failure η is substituted for X in Eq. (1) and e for $1/\alpha$, and where $F=1-S$, Eq. (1) can be written

$$F=1-\exp\{-(\eta/\beta)^e\} \tag{2}$$

The characteristic life of β is the 63.2% failure life of the population distribution.

Weibull stated that the probability of survival S could be expressed as

$$\ln\frac{1}{S}\sim\tau^c\eta^e V \tag{3}$$

where V is the volume representation of stress concentration referred to herein as stressed volume, τ is the critical shearing stress, and c is an exponent denoting a stress-life relation where

$$\eta\sim\tau^{-c} \tag{4}$$

The values of c and e can be determined experimentally.

The effect of stressed volume can be illustrated where two specimens of stress volumes V_1 and V_2, respectively, are subjected to equal stress τ. If η_1 is determined at a probability of survival S_1, then the probability of survival S_2 for V_2 for the life is given by the following expression

$$S_2=S_1^{V_2/V_1} \tag{5}$$

where $V_2>V_1$.

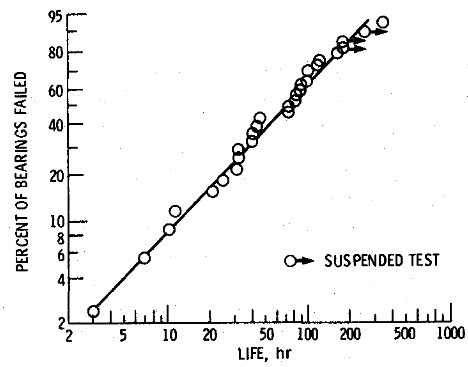

Fig. 1 Weibull distribution of bearing fatigue life for 140-mm bore-size angular-contact ball bearing. MIL-L-7808 lubricant; thrust load, 9500 lb; speed, 10,000 rpm; temperature, 121°C (250°F).

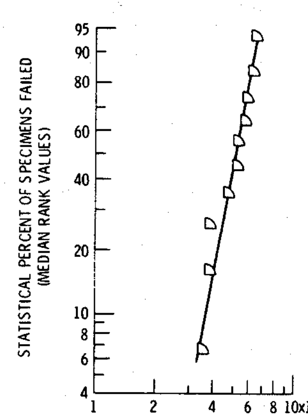

Fig. 2 Weibull distribution for bond shear strength for coating of spray powder on stainless steel.[11]

For the same probability of survival, the specimens with the larger stressed volume will have the lower life. This principle was applied to successfully normalize rolling-element fatigue data by Carter[9] and Zaretsky.[10] This principle was applied by Grisaffe[11] in a Weibull analysis of shear bond strength of plasma-sprayed alumina coatings on stainless steel (Fig. 2). Grisaffe showed that the calculated mean bond strength decreased with increasing test area in accordance with Eq. (5) (Fig. 3). Further, the stress at which no specimens failed could be determined by restating Eq. (1) as follows:

$$F_x = 1 - \exp\left[-\left(\frac{X - X_\mu}{X_\beta}\right)^e\right] \tag{6}$$

where the parameters are now defined as:

F_x = statistical percent of specimens which, when tested at one set of conditions, fail at given stress or lower
X = stress
X_μ = stress below which no specimens failed
X_β = characteristic strength
e = Weibull slope

Lundberg-Palmgren Theory

Lundberg and Palmgren were contemporaries of Weibull. A problem existed in the rolling-element bearing industry: how to establish the lives of these bearings short of extensive testing. Taking the Weibull approach a step further, Lundberg and Palmgren[3] took Eq. (3) and added another element, the depth to the critical maximum shearing stress z, where

$$\ln\frac{1}{S} \sim \frac{\tau^c \eta^e V}{z^h} \tag{7}$$

The rationale for including the depth z was that the initiation of a fatigue crack occurred at the depth z and that the distance the crack needs to travel to the surface until a spall (pit) occurs is equivalent to a critical crack length. In other words, the greater the distance the maximum shearing stress is below the surface, the longer it takes for a fatigue crack to propagate to the surface, and the longer the fatigue life. Lundberg and Palmgren took the rolling-bearing geometry, kinematics, stress theory of Hertz[12] and Thomas and Hoersch[13] and incorporated them into Eq. (7) to obtain a series of equations that relate the rolling-element fatigue life of a bearing to the applied load, where the resultant life would be in millions of inner-race revolutions. When the speed of the bearing is known, the life in hours can be determined. Lundberg and Palmgren further refined their approach to bearing-life prediction to include a fictitious load designated C, the dynamic load capacity. The dynamic load is the theoretical load which, when applied to the bearing, would result in a life of one million inner-race revolutions and where

$$C = P^n \sqrt{L_{10}} \tag{8}$$

P = applied load; L_{10} = 10% life, life at a 90% probability survival, inner-race revolutions; and n = load-life exponent, usually taken as 3. Equation (8) can be written

$$L_{10} = (C/P)^n \tag{9}$$

Hence, the predicted life of a bearing can be determined by knowing C and P. Conversely, by experimentally determining the L_{10} life, C can be calculated.

The Lundberg-Palmgren theory is now an international standard used by every manufacturer of bearings around the world.[14] The theory has been applied to other machine elements and mechanical transmission systems.[15-28] Table 1 is a list of applications of the Lundberg-Palmgren theory to life prediction.

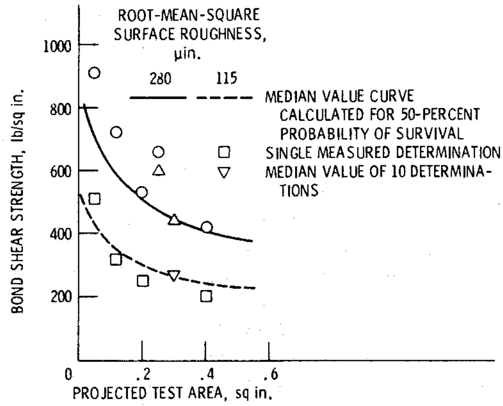

Fig. 3 Effect of projected test area on measured and calculated bond shear strength of coatings at two surface roughnesses.[11]

Table 1 Lundberg-Palmgren[3-5] fatigue life analysis (based upon 1939 Weibull theory)[1,2]

Analysis	Year published	Ref.
Bearings	1947	3
Spur gears	1975	15
Helical gears	1976	17
Restored and refurbished bearings	1977	19
Load-life relation	1978	20
Toroidal drive (traction)	1976	18
Simplified life analysis for traction drives	1981	21
Nasvytis drive	1981	22
Optimization of multiroller traction drive	1982	24
Spiral bevel gears	1982	26
Planetary assemblies	1983	25
Rotary beam, torsion & tension-compression	1985	28
Transmission assemblies	1986	36

System Life and Reliability

The life and reliability of a system is based on the lives and reliabilities of all its components. With the life of the individual components determined using Lundberg-Palmgren, the probability of survival of the entire system is as follows:

$$S_T = S_1 \cdot S_2 \cdot S_3 \ldots S_n \tag{10}$$

The system life-reliability equation can be written as follows

$$\ln\frac{1}{S_T} = \ln\frac{1}{0.9}\left[\left(\frac{L}{L_1}\right)^{e_1} + \left(\frac{L}{L_2}\right)^{e_2} + \left(\frac{L}{L_3}\right)^{e_3} \ldots + \left(\frac{L}{L_\beta}\right)^{e_n}\right] \tag{11}$$

where $L_1, L_2, L_3 \ldots L_n$ are the lives of each component of the system at a 90% probability of survival. The system life L can be determined at each system probability of survival S_T. For a 90% probability of survival is

$$1 = \left(\frac{L}{L_1}\right)^{e_1} + \left(\frac{L}{L_2}\right)^{e_2} + \left(\frac{L}{L_3}\right)^{e_3} \ldots + \left(\frac{L}{L_n}\right)^{e_n} \tag{12}$$

Mission Life

A system does not usually operate at one constant load in actual service. Miner's rule is used to sum fatigue damage of a mission profile consisting of loads and time at loads. For a given probability of survival, the mission life for the system L_m is

$$L_m = \left(\frac{t_a}{L_a} + \frac{t_b}{L_b} + \frac{t_c}{L_c} + \frac{t_d}{L_d}\right)^{-1} \tag{13}$$

where t_a, t_b, t_c, and t_d are the fractions of the total time at loads P_a, P_b, P_c, and P_d, respectively, and L_a, L_b, L_c and L_d are the system lives at a given probability of survival at loads P_a, P_b, P_c, and P_d, respectively. A Weibull plot or distribution can be constructed using this method by determining the mission life at varying probabilities of survival.

This method was used by Lewicki et al. [28] to determine the fatigue life of an Allison T56/501-D22A gearbox (Fig. 4). The theoretical mission profile of loads and time at loads for the gearbox is given in Table 2. These data represent a typical mission profile, even though the actual profile varies from mission to mission. The loads on each component were determined from these data.

Bearing lives were determined using Lundberg-Palmgren theory. The lives for all the cylindrical roller bearings were calculated using the computer program CYBEAN.[29] The lives of the prop thrust ball bearing were calculated using the computer program SHABERTH.[30] The lives for the planet spherical roller bearing were calculated using the computer program SPHERBEAN.[31] The rollers and raceways of all bearings except the front and rear pinion bearings are made from consumable-electrode vacuum-melted (CEVM) AISI 52100 or AISI 9310 steel. They were all given a combined material and material-processing life-adjustment factor of 6 (2 for material and 3 for material processing[32]). The rollers and raceways for the front and rear pinion bearings are made from vacuum-induction-melted, vacuum-arc-remelted (VIM-VAR) AISI M-50 steel. A combined material and material-processing life-adjustment factor of 12 was chosen for the front and rear pinion bearings.[33] For simplicity, the material life-adjustment factor will refer to a combined material and material-processing life-adjustment factor.

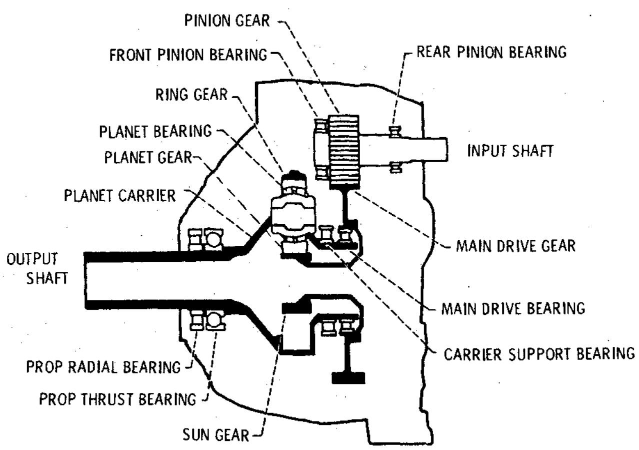

Fig. 4 Allison T56/501 reduction gearbox.

Table 2 Mission profile of Allison T56/501 gearbox used for analytic predictions

Mission segment	Percent time of segment	Prop shaft power, kW (hp)	Prop shaft moment, N·m (in-lb)
Takeoff	2.84	3132 (4200)	5875 (52,000)
Climb	17.02	2461 (3300)	5875 (52,000)
Cruise	68.08	1516 (2033)	5197 (46,000)
Descent	12.06	945 (1267)	3728 (33,000)

Due to the similarity in the fatigue failure mechanism between gears and rolling-element bearings made from high-strength steel, the Lundberg-Palmgren life model for bearings has been adapted to predict gear life.[15-17,20,23] Experimental research of AISI 9310 steel spur gears has shown gear fatigue life to follow the Weibull failure distribution with an average Weibull exponent of about 2.5.[20] A generalized life-reliability equation may be written for each of the gears in the gearbox. For each gear,

$$\log\frac{1}{S_G} = \log\frac{1}{0.9}\left(\frac{L}{L_{10_G}}\right)^{e_G} \tag{14}$$

where

$$L_{10_G} = \frac{N^{-1/e_G}\eta_{10_t}}{k} \tag{15}$$

for all gears except the planet gear and

$$L_{10_G} = \frac{N^{-1/e_G}(\eta_{10_{t1}}^{-e_G} + \eta_{10_{t2}}^{-e_G})^{-1/e_G}}{k} \tag{16}$$

for the planet gear and

$$\eta_{10_t} = a_t(C_t/P_t)^{n_G} \tag{17}$$

where

$$C_t = Bw^{0.907}\Sigma\rho^{1.165}l^{-0.093} \tag{18}$$

η_{10_t} is the number of millions of stress cycles for which one particular tooth of a gear has a 90% probability of survival. η_{10_t} can be determined using Eq. (17), where C_t is the basic dynamic capacity of the gear tooth, P_t the normal tooth load, n_G the load-life exponent based on experimental data (equal to 4.3), and a_t the life-adjustment factor. C_t can be determined using Eq. (18) where B is the material constant based on experimental data and found to be $\sim 1.39 \times 10^8$ when SI units are used (newtons and meters) and 21,800 when English units are used (pounds and inches) for AISI 9310 steel spur gears and where w is the tooth-face width, $\Sigma\rho$ the curvature sum at the start of single-tooth contact, and l the length of the involute surface during single-tooth contact. L_{10_G} is the life of the gear (all teeth) in millions of input shaft revolutions in which 90% will survive. L_{10_G} can be determined by Eqs. (15) or (16), where N is the total number of teeth on the gear, e_G the Weibull exponent (2.5), and k the number of load cycles of a gear tooth per input shaft revolution.

For all the gears except the planet gear, each tooth will see contact on only one side of its face for a given direction of input shaft rotation. Each tooth on a planet gear, however, will see contact on both sides of its face for a given direction of input shaft rotation. One side of its face will contact a tooth on the sun gear and the other side of the face will contact a tooth on the ring gear. Equation (16) takes this into account. $\eta_{10_{t1}}$ is the millions of stress cycles for a 90% probability of survival of a planet tooth meshing with the sun gear, and $\eta_{10_{t2}}$ is the millions of stress cycles for a 90% probability of survival of a planet tooth meshing with the ring gear.

Figure 5 shows the life distribution for the individual bearings and gears for the gearbox using the above methods, as well as the system life. The system life and reliability is based on the lives and reliabilities of all the bearings and gears. Using the subscripts B for the bearings and G for the gears, the probability of survival of the gearbox S_T is obtained from

Eq. (10), where

$$S_T = S_{B1} \cdot S_{B2} \cdot S_{B3} \cdot S_{B4} \cdot S_{B5} \cdot S_{B6} \cdot S_{B7}^m \cdot S_{G1} \cdot S_{G2} \cdot S_{G3} \cdot S_{G4}^m \cdot S_{G5} \quad (19)$$

where m is the number of planets, and the subscript T designates transmission assembly. Taking the logarithm of the inverse of Eq. (19) and with Eq. (11), the generalized system life-reliability equation is

$$\log\frac{1}{S_T} = \log\frac{1}{0.9}\left[\left(\frac{L}{L_{10_{B1}}}\right)^{e_B} + \left(\frac{L}{L_{10_{B2}}}\right)^{e_B} + \left(\frac{L}{L_{10_{B3}}}\right)^{e_B} + \left(\frac{L}{L_{10_{B4}}}\right)^{e_B} + \left(\frac{L}{L_{10_{B5}}}\right)^{e_B} + \left(\frac{L}{L_{10_{B6}}}\right)^{e_B} + m\left(\frac{L}{L_{10_{B7}}}\right)^{e_B} + \left(\frac{L}{L_{10_{G1}}}\right)^{e_G} + \left(\frac{L}{L_{10_{G2}}}\right)^{e_G} + \left(\frac{L}{L_{10_{G3}}}\right)^{e_G} + m\left(\frac{L}{L_{10_{G4}}}\right)^{e_G} + \left(\frac{L}{L_{10_{G5}}}\right)^{e_G}\right] \quad (20)$$

where the probability of survival for the complete gearbox S_T is a function of millions of input shaft revolutions L and the lives at a 90% probability of survival of each bearing and gear in terms of millions of input shaft revolutions. The Weibull exponent e_B for the bearings used was 1.1.

For a given load on the gearbox, the lives of each bearing and gear will be constants and can be determined. Using Eq. (20), the system life for a given probability of survival can be calculated using an iterative process. A curve can be plotted on Weibull coordinates using a variety of S's and corresponding L's. These curves may not be straight lines due to the different slopes for bearings and gears. For any S, the system life is always less than the life of the shortest-lived component at the same S.

Figure 6 shows the system or mission lives of the gearbox calculated with and without life-adjusted factors and compared with actual field experience. The lubrication life-adjustment factor is based on the ratio of the elastohydrodynamic (EHD) film thickness to composite surface roughness.[32] The best correlation with field experience occurred using the material life-adjustment factors without the lubricant life-adjustment factor.

Component Replacement Rate

The Weibull analysis is a valuable tool for predicting the life of components or systems. However, the analysis describes the failure rate in field service only if all components were put into service at the same time and if failed components were not replaced. Since a certain number of components must be kept in operation, failure rates with replacements are of interest. As an example, if there are 10,000 bearings in the field, the user or manufacturer must know how many spare bearings will be needed over a given period in order to keep 10,000 bearings in operation at all times. Over the interval of service operation, it may take 15,000 replacements to keep all 10,000 bearings running. The Weibull analysis can never exceed 100%, but field failures can and often do exceed 100%.

The cumulative probability of failure for the first time, assuming constant service conditions, is a function of the length of time the bearing has been running. Failure is the complementary function to survival according to the following relation:

$$F(t) = 1 - S(t) \quad (21)$$

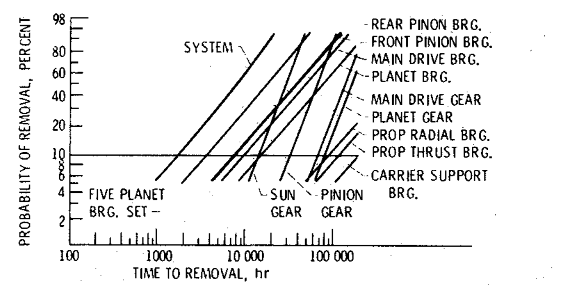

Fig. 5 Analytically predicted system and component fatigue mission lives. Combined material processing life-adjustment factors are used for bearings; gear material is baseline material, no factors needed; and no lubrication lfe-adjustment factors are used.[28]

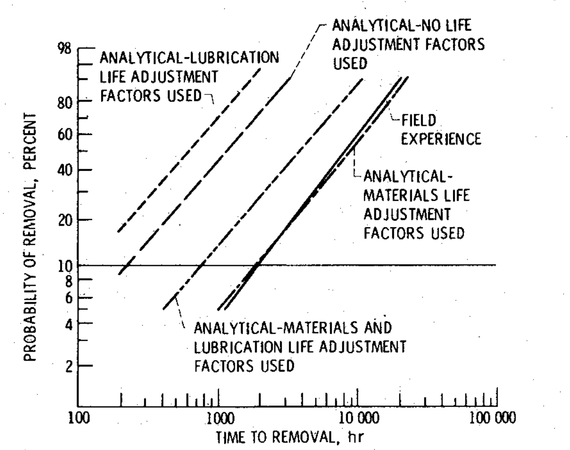

Fig. 6 Turboprop reduction gearbox fatigue lives (field experience) compared with analytically predicted system mission lives. Material factor implies combined material and material processing factor.[28]

The probability density function for failure is

$$f(t) \equiv \frac{dF}{dt} \quad (22)$$

The instantaneous probability of failure for the first time in any time interval from t to $t+\Delta t$ is then given by

$$P(\text{first failure in } \Delta t \text{ interval}) = f(t)\Delta t \quad (23)$$

Using renewal theory,[34,35] the probability of having to replace a bearing in the field is written as follows:

$$P = r(t)\Delta t \quad (24)$$

where P denotes a probability of replacement and $r(t)$ is called the renewal density. The renewal density is calculated from the following summation:[34]

$$r(t) = \sum_{k=1}^{X} f_k(t) \quad (25)$$

where $f_k(t)$ is the k-fold convolution of f with itself and is computed by the following recurrence relation:[34]

$$f_{k+1}(t) = \int_0^t f_k(T) f_1(t-T)\,dT \qquad (f_1 \equiv f) \quad (26)$$

The expression $f_k(t)\Delta t$ gives the probability of a kth failure occurring in the time interval t to $t+\Delta t$. Since a failure can occur for any value of k, it follows that the renewal density function should be defined as the sum over all the f_k's.

The total number of replacements made during the first t units of time is obtained by integrating the renewal density function as follows:

$$R(t)=\int_0^t r(T)\mathrm{d}T \tag{27}$$

For a group of bearings that have been operating for some time with failures occurring and replacements being made, it is also important to know the MTBF (mean time between failures). According to the renewal density theorem,[35]

$$\lim_{t\to\infty} r(t)=\frac{1}{\int_0^\infty tf(t)\mathrm{d}t}\equiv\frac{1}{\gamma} \tag{28}$$

Therefore, the mean time between failures is calculated as

$$\mathrm{MTBF}=\frac{\gamma}{\text{total number of bearings}} \tag{29}$$

Reference 19 reports the use of a computer program to evaluate Eq. (25) for rolling-element bearings. A Weibull slope of 1.1 was assumed. Figure 7 shows plotted results that give the renewal density function for a case in which the failed bearings are removed from service and replaced with new or restored bearings. For comparison, the probability density for failures with no replacement (Weibull density function) is plotted also. The area under the curves represents the probability of failure.

The functions plotted in Fig. 7 were numerically integrated and are shown plotted in Fig. 8. These are the cumulative functions for renewal or failure. The cumulative renewal functions indicate that 100% replacement bearings will have been needed by the time 8 L_{10} intervals have elapsed. By comparison, at the time of 8 L_{10}, the Weibull distribution shows that only 65% of the original bearings will have failed. The difference in total failures would be due to replacement bearings failing.

The MTBF from Eq. (29) can be obtained from Fig. 7. The MTBF is the inverse of the probability density for failure. As an example a probability density for failure of 0.12 would give a MTBF value of 8.3 L_{10}.

Fatigue Life Modeling

The Weibull and Lundberg-Palmgren analyses have been primarily applied to high cycle fatigue with the material subjected to a Hertzian stress field. However, as indicated in Ref. 11, the Weibull analysis can be applied to other types of durability problems. What is important is that a material element only knows the state of stress it is subjected to and not whether it is in a bearing, gear, shaft, compressor, or turbine. The crack propagation time in high cycle fatigue is generally a small fraction of the total time to failure. In low cycle fatigue, the crack propagation time is generally a significant fraction of the time to failure. In either case, the end result is total fracture of the component, making it no longer useful for its intended purpose. If the failure distribution is within standard statistical ranges, then it can be represented by the Weibull analysis. Hence, the analysis should be blind to high or low cycle fatigue. This was recognized by Ioannides and Harris.[36] Using Weibull and Lundberg-Palmgren, they introduced a "fatigue limiting stress" and integrated the computed life of elemental stress volumes to predict life. This approach leads to a method of applying Lundberg-Palmgren life-prediction techniques to other components besides those subject to Hertzian loading. It also allows an investigator to use the results of coupon testing to predict the life of complex shaped components subjected to non-Hertzian cyclic stressing. Ioannides and Harris[36] applied their analysis successfully to rotating beam fatigue (Fig. 9), flat beams in reversed bending (Fig. 10), and beams in reverse torsion. Based on their approach or a modification thereof, design procedures for structures subjected to fatigue loading can be formulated that allow for finite life determination in the initial design stage with reasonable statistical certainty.

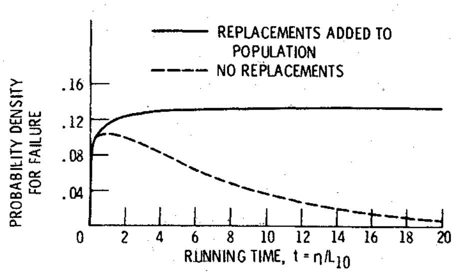

Fig. 7 Renewal density functions compared for rolling-element bearings. Also shown is probability density for failure of new bearing assuming no replacements. Area under curves represents probability of failure.

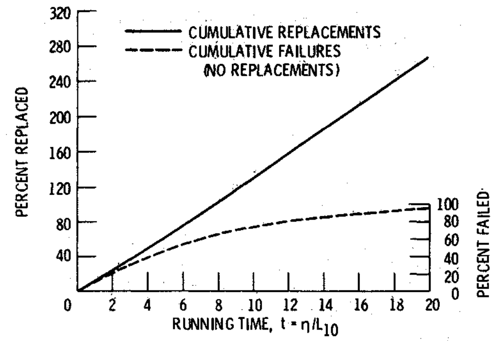

Fig. 8 Cumulative renewal and failure for rolling-element bearings.

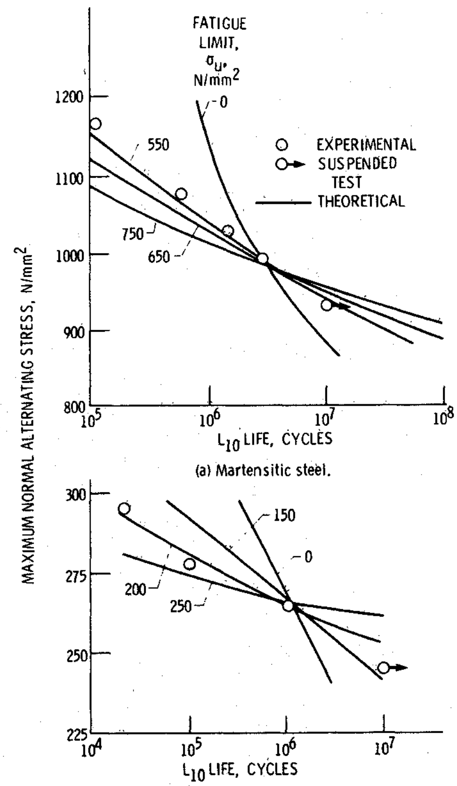

Fig. 9 Comparison between experimental and predicted stress-life relation for AISI 52100 steel rotating beam specimens for calculated values of fatigue limit σ_u.[36]

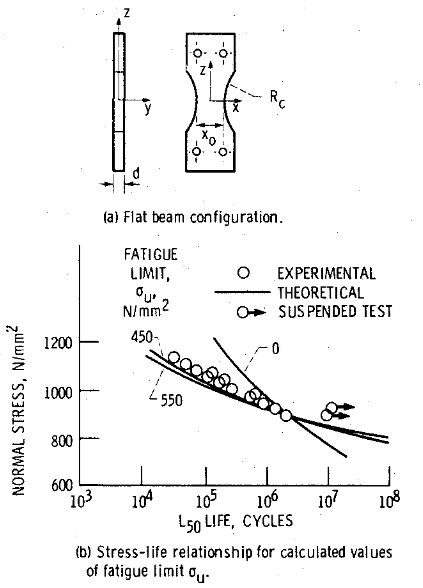

Fig. 10 Comparison between experimental and predicted stress-life relation for flat beam specimens made from bearing steel and tested in reverse bending.[36]

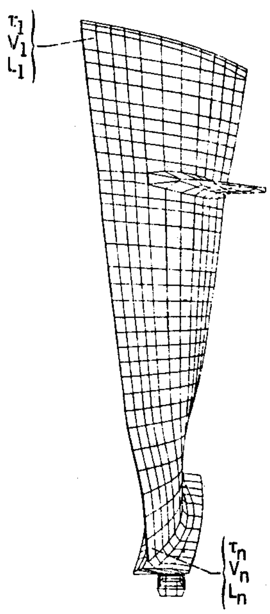

Fig. 11 Finite element analysis of turbine engine fan blade.

Design of Complex Structures

In recent years, finite element stress analysis of complex structures subjected to thermal and mechanical loading has reached a high degree of sophistication and reliability. Now computers perform calculations in mere seconds and minutes for problems that only a short time ago would take hours or be impossible to perform. An example of a turbine blade analyzed using finite element analysis is shown in Fig. 11. Each element of material has associated with it a stress. For design purposes, this stress is within certain established stress or strain limits. The ultimate life of the component is usually based on empirical calculations or extrapolation from field experience. The results are at best highly speculative. By subjecting the component to expensive product improvement programs (PIP) and by "make-and-break" techniques, component lives over a product's lifetime can usually be extended to the useful life of the product.

The key to cost-effective design is to be able to maximize component life and minimize cost at the completion of the final design stage without the need for extensive testing or field experience. It is proposed that this objective can be accomplished as follows:

1) Having determined the stress of each elemental volume of a component using finite element analysis, it is possible to establish a life of the elemental stressed volume using either Eqs. (3) or (7).

2) Using Eq. (12), it is then possible to establish the life of the entire component.

3) Carrying the process a step further, applying the principle of Eq. (8), a "dynamic" capacity of the component can be determined.

4) Once the dynamic capacity is determined, the mission life of a component or system can be established using Eq. (13).

5) Using renewal theory [Eqs. (25) and (29)] the number of replacements required over a system's useful life and the MTBF can be determined.

By design iteration, design life can be maximized and cost minimized. Further, with failure rates known, production guidelines can be set before a product enters the marketplace.

In the area of bearing technology, vast technological improvements occurred between initial publication of Lundberg-Palmgren (1947) and contemporary development. These technological improvements were factored into the analysis by the use of life-adjustment factors,[32] in which the adjusted life L_A at a 90% probability of failure is

$$L_A = (D)(E)(F)(G)(H)(C/P)^n \tag{30}$$

where the life-adjustment factors are:

D = materials factor
E = processing factor
F = lubrication factor
G = speed effect
H = misalignment factor

A similar approach to the design of power-transmitting shafts was taken by Loewenthal.[37] The Loewenthal approach is based on a fatigue-limiting stress which, in turn, is based on combined torsion, bending, and axial loading. The fatigue-limiting stress is modified by ten factors that, in principle, are in part incorporated in the Weibull and Lundberg-Palmgren analyses. Those factors outside the analysis can be based on experimental coupon fatigue data.

Most machine elements are subjected to their maximum design load only for a very small fraction of their mission life cycle. Hence, by designing to a life criterion and a mean cubic load rather than a fatigue-limiting stress, it is possible to reduce the size of the machine element without reducing system reliability.

Conclusion

A generalized methodology to structure life prediction, design, and reliability based on a fatigue criterion is advanced. The life-prediction methodology is based in part on the work of W. Weibull and of G. Lundberg and A. Palmgren. The approach incorporates the computed life of elemental stress volumes of a complex machine element to predict system life. The results of coupon fatigue testing can be incorporated into the analysis, allowing for life-prediction and component- or structural-renewal rates with reasonable statistical certainty.

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